Radiation From Panels as a Source of Airframe Noise

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AS propulsive noise has been systematically decreased for aircraft, the noise of the airframe itself has become of some concern, particularly for large aircraft with flaps, spoilers, landing gear, etc., deployed during landing approach.[1-4] A number of purely fluid mechanical mechanisms for airframe noise have been considered. Here we consider, in a simplified fashion, the possibility of far field sound radiation due to vibrating structural panels, which themselves are excited by a near field (random) sound source such as a turbulent boundary layer or separated flows due to flap or spoiler deployment.

The two parts of the problem: 1) excitation of panels by random (near field) pressures; and 2) far field radiation due to vibrating panels, have been extensively studied. For our purposes we shall use for part 1) the simple, single structural mode approximation with a uniform spatial pressure distribution as given by Miles.[5,6] For part 2) we use the well-known results for radiation from a planar piston, as described by Morse and Ingard.[7] For simplicity, panel curvature is neglected.

In Miles' model, it is assumed that the structural response is dominated by a single mode and hence

$$w=q(t)\,\psi(x,y)$$

where w = panel displacement, q = generalized coordinate, ψ = mode shape.

From Miles' model, it is determined that[5,6]

$$\bar{q}^2=(\pi/4)\,[\Phi_p(\omega_N)/M^2\omega_N{}^3\zeta_N]\,[\iint\psi\,\mathrm{d}x\mathrm{d}y]^2 \qquad (1)$$

where $M\equiv[\rho_p t\iint\psi^2\mathrm{d}x\mathrm{d}y]$ is the generalized mass, ω_N = model natural frequency, ζ_N = modal damping, $\Phi_p(\omega_N)$ = power spectra of near field for $\omega=\omega_N$, ρ_p = panel density, and t = panel thickness. It is assumed that the panel is lightly damped and hence the reponse is dominated by the panel resonant response near $\omega=\omega_N$.

From Ref. 7, the far field pressure, p_F, at a distance r perpendicular to the panel surface is given (for a sinusoidally oscillating surface) by

$$p_F=(i\omega\rho/2\pi r)\;V\exp^{[i\omega(t-r/c)]} \qquad (2)$$

where ρ = fluid density and V = fluid volume rate of change due to panel motion $=[\iint\psi\mathrm{d}x\mathrm{d}y]\,(i\omega q)$ for our problem. Combining Eqs. (2) and (1), assuming the dominate response is still near $\omega=\omega_N$, and using the concept of a transfer function, one obtains

$$\bar{p}_F^2=[\Phi_p(\omega_N)\omega_N/\zeta_N]\;(\rho ab/\rho_p tr)^2K \qquad (3)$$

where

$$K\equiv\frac{\pi}{4(2\pi)^2}\,[\iint\psi\,\frac{\mathrm{d}x}{a}\,\frac{\mathrm{d}y}{b}\,]^4/[\iint\psi^2\,\frac{\mathrm{d}x}{a}\,\frac{\mathrm{d}y}{b}\,]^2$$

From Eq. (3) we see that large (a), thin (t), light (ρ_p) panels with small damping (ζ_N) are the most efficient radiators of sound. For a clamped rectangular plate of length a and width b, we take

$$\psi\cong[1-\cos 2\pi x/a]\,[1-\cos 2\pi y/b]$$

and thus $K=(81\,\pi)^{-1}$.

Consider the following numerical example of an aluminum panel with $E=10^7$ psi, $\rho_p=0.1$ lb/in.^{3}, $a=24$ in, $b=18$ in., $t=0.05$ in., and $\zeta_N=0.01$. Hence $\omega_N=37$ cps and from Ref. 8, Fig. 12 (which provides a representative turbulent boundary-layer pressure spectra), $\Phi_p=0.00305$ psi^2/cps. Also, for sea-level conditions, $\rho a/\rho_p t=0.21$. Finally assume $b/r=0.005$. Then $[\bar{p}_F{}^2]^{1/2}=97$ db.

This is strikingly high sound level (although at a rather low frequency); however, it is only intended as an illustration of how to use the suggested method. Clearly, the sound level may be higher or lower, depending on the parameter values chosen. The ones selected here are thought to be representative.

More importantly, there are simplifications in the model which may be untenable under some circumstances.

1) We have assumed the near field pressure is spatially uniform. A simple approximate correction can be made[9] by multiplying the far field pressure with the following factor

$$\left\{\frac{4L_xL_y\iint\psi^2\mathrm{d}x\mathrm{d}y}{[\iint\psi\mathrm{d}x\mathrm{d}y]^2}\right\}$$

where L_x, L_y are correlation lengths in the x,y directions ($L_x\ll a$, $L_y\ll b$). For our assumed ψ and taking $L_x/a=L_y/b=0.01$, the sound level would be reduced by 10 db.

2) The radiation of sound to the far field provides additional damping to the panel and will reduce thereby the far field pressure. For the low velocities associated with landing aircraft this should not be substantial, but there are methods of various sophistication which allow one to compute an equivalent ζ_N.[10-12]

3) The behavior at large frequency would require more attention if the panel damping is not sufficiently small. Note that 1) the transfer function relating p_F to w is proportional to ω^4, 2) the transfer function relating w to the near field pressure is proportional to ω^{-4} for large ω and hence, 3) the total transfer function approaches a constant for large ω. Thus Φ_p must decrease with frequency more rapidly than ω^{-1} for large frequency to obtain a finite far field pressure. The more rapidly the better for the present simplified analysis, of course.

Finally, it is noted that there is no difficulty, in principle, in extending the analysis to include 1) many structural modes and 2) radiation damping for the panel. Indeed Davies[13] and Aupperle and Lambert[14] have made substantial contributions in this area. The purpose of the present note is to suggest that it is desirable to develop such an analysis systematically and apply it to airframe noise estimation.

For example, if one extends the present analysis to many modes, one simply replaces Eq. (1) (assuming small damping in any one mode) by

$$\bar{q}_m{}^2=\frac{\pi}{4}\sum_m\frac{\Phi(\omega_m)}{M_m{}^2\omega_m{}^3\zeta_m}\,[\iint\psi_m\mathrm{d}x\mathrm{d}y]^2 \qquad (4)$$

where m is the modal subscript and Eq. (3) becomes

$$\bar{p}_F{}^2=\left[\frac{\rho ab}{\rho_p tr}\right]^2\sum_m\frac{\Phi_p(\omega_m)\omega_m K_m}{\zeta_m} \qquad (5)$$

where

$$K_m=\frac{1}{16\pi}\,[\iint\psi_m\,\frac{\mathrm{d}x}{a}\,\frac{\mathrm{d}y}{b}\,]^4/[\iint\psi_m{}^2\,\frac{\mathrm{d}x}{a}\,\frac{\mathrm{d}y}{b}\,]^2$$

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Equation (5) may be a near optimum compromise between accuracy and simplicity with the estimation of damping ζ_m the critical factor in most applications.[10-12]

There are other effects which could be included in a more refined analysis. For example, the motion of the panel may change the near field turbulent-flow pressure fluctuations which excite the panel. However, the effect is likely to be small under most circumstances and, in any event, our present meager knowledge does not justify its inclusion in a simplified analysis.

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Practical Aspect of the Generalized Inverse of a Matrix

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THE rules of matrix algebra make it possible to write m equations with n unknowns x in the form of a matrix equation

$$[A]_{m,n}\{x\}_{n,1}=\{y\}_{m,1} \tag{1}$$

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In the following discussion it is assumed that the equations represented by Eq. (1) are linearly independent. If $m=n$ there is a unique solution for $\{x\}$ in terms of the right-hand side $\{y\}$

$$\{x\}=[A]^{-1}\{y\} \tag{2}$$

The solution for $\{x\}$ is not affected by multiplication of any equation in Eq. (1) by a finite nonzero number.

If $m>n$ there are more equations than unknowns: the problem is over determined. In the so-called least-squares approach, a unique solution can be found by minimizing

$$\sum_{i=1}^{m}\epsilon_i^2$$

where ϵ is an "error," the difference between the left-hand and right-hand sides of each individual equation in Eq. (1)

$$\{\epsilon\}_{m,1}=[A]_{m,n}\{x\}_{n,1}-\{y\}_{m,1} \tag{3}$$

The solution is given by

$$\{x\}_{n,1}=[[A^T]_{n,m}[A]_{m,n}]^{-1}[A^T]_{n,m}\{y\}_{m,1} \tag{4}$$

where $[A^T]$ is the transpose of $[A]$.

For a given matrix $[A]$, Eq. (4) defines a unique solution for $\{x\}$. The solution, however, is affected by multiplying one or more of the equations in Eq. (1) by arbitrary numbers.

The preceding formulations are in general use and well understood. The purpose of this Note is to add to the understanding of the case that $m<n$. That is the case with fewer equations than unknowns: the problem is underdetermined.

From the general theory of equations it is known that if $m<n$, Eq. (1), does not lead to a unique solution $\{x\}$. In fact, an infinite number of solutions $\{x\}$ are possible. To make it possible to define a unique solution $\{x\}$ $(n-m)$ equations must be added to Eq. (1), or, equivalently, some constraints must be put on the relation between the elements of $\{x\}$.

In the literature, a generalized inverse of $[A]_{m,n}$ $(m<n)$ is given the notation $[A]^+_{n,m}$. Parallelling Eq. (2) it defines $\{x\}$ in terms of $\{y\}$ as follows

$$\{x\}_{n,1}=[A]^+_{n,m}\{y\}_{m,1} \tag{5}$$

An expression for $[A]^+$ can be found in many places in the literature.[1-8] This author, however, has failed to find a specific reference to the additional constraints on the elements of $\{x\}$ that make it possible to determine a unique matrix $[A]^+$.

In the following derivation it is shown that an expression for $[A]^+$ identical to the one found in the literature is found if the following constraint on $\{x\}$ is applied

$$\{x\}_{n,1}=\alpha_1\{a_1\}_{n,1}+\alpha_2\{a_2\}_{n,1}+\ldots\alpha_m\{a_m\}_{n,1} \tag{6}$$

where $\{a_1\}$, $\{a_2\}$,...$\{a_m\}$ are the transposes of the m rows of $[A]_{m,n}$. Equation (6) can be written as

$$\{x\}_{n,1}=[A^T]_{n,m}\{\alpha\}_{m,1} \tag{7}$$

Substituting Eq. (7) into Eq. (1) leads to

$$[A]_{m,n}[A^T]_{n,m}\{\alpha\}_{m,1}=\{y\}_{m,1} \tag{8}$$

Thus, Eq. (1) with n unknowns x has been transformed into Eq. (8) which contains m equations with m unknowns α.

Solving Eq. (8) for $\{\alpha\}$ gives

$$\{\alpha\}_{m,1}=[[A]_{m,n}[A^T]_{n,m}]^{-1}\{y\}_{m,1} \tag{9}$$